Research Internship Proposal
Proof Systems for Intuitionistic Non-Normal Modal Logic

Scientific Context

Intuitionistic logic is an alternative to classical logic. It is based on a constructive conception of truth: a proposition is "true" if and only if it admits a proof. Modal intuitionistic logic has an important number of applications in computer science, notably in type theory and in λ-calculus (via the Curry-Howard isomorphism), but also in verification (constructive proofs of program properties, verification of protocols. . .). There even are applications in spatial reasoning (logic of topological spaces).

Modal intuitionistic logics extend intuitionistic logic with two modalities, namely $\Box$ (Necessity) and $\Diamond$ (Possibility). There are multiple definitions of modal logics, because:

- the semantic interpretation of $\Box$ and $\Diamond$ can be defined in several ways
- but also, the basic axioms relating the modal operators with the usual propositional connectives can be chosen in multiple ways.

These various logics have multiple applications. For this internship, the modal intuitionistic logic of interest is the intuitionistic non-distributive modal logic studied by Wijesekera [4] and Kojima [2]. In this logic, the classical equivalence $\Diamond(A \vee B) \equiv \Diamond A \vee \Diamond B$ is not an axiom. This logic (and its extensions) have been studied in the context of dynamic concurrent logic, of hardware verification, of typed λ-calculus and in security (access control). It also has an interest for epistemic reasoning. Because of the non-distributivity of $\Diamond$ over $\vee$, this logic is similar to non-normal modal logics characterized by neighborhood semantics. Such a neighborhood semantics was recently defined by Kojima [2].

Internship Objectives:
This internship aims at defining correct and complete proof systems for this logic, under the form of labeled sequent and tableaux calculi based on neighborhood semantics, following the approaches of [1,3]. The calculi should lead to decision procedures for this logic.

Administrative Information

- *Supervisors* : Charles Grellois and Nicola Olivetti, Aix-Marseille Université, Laboratoire LSIS-LIS, équipe LIRICA
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- *Internship Location*: Laboratoire LSIS (LIS) Marseille

- *Funding*: The internship can be funded by the ANR/PRCI TICAMORE project (see `https://ticamore.logic.at/`)
- *Expected Duration* : 4 to 5 months
- *Time Period*: the internship should begin between Feb 1st, 2018 and March 15th, 2018, and end before July 31th, 2018.

References

1. M. Girlando, S. Negri, N. Olivetti, V. Risch: The Logic of Conditional Beliefs: Neighbourhood Semantics and Sequent Calculus. Advances in Modal Logic 2016: 322-341
2. K. Kojima: Relational and neighborhood semantics for intuitionistic modal logic. Reports on Mathematical Logic 47: 87-113 (2012)
3. S. Negri: Proof theory for non-normal modal logics: The neighbourhood formalism and basic results, IfCoLog, 4 (2017), pp. 1241-1286
4. D. Wijesekera: Constructive modal logics I, Annals of Pure and Applied Logic 50 (1990), pp. 271–301.