Research Internship Proposal
Higher-Order Model-Checking and Linear Logic

Scientific Context

The main idea of model-checking is to abstract a program into a model, by omitting information about its behavior in order to simplify reasoning. Then, logical specifications are formulated on this model, and the goal is to determine automatically whether the model of the program satisfies the specification.

In higher-order model-checking (HOMC), we consider abstractions of functional programs. They are modeled by infinite trees of actions. These trees are not regular in general: they can not be represented as the unfoldings of finite graphs. However, they can be represented using higher-order recursion schemes (HORS), a grammar-like formalism which turns out to be equivalent to a simply-typed λ-calculus with fixpoints. The logic we want to decide over these infinite trees – finitely represented by HORS – is monadic second-order logic (MSO), whose automata-theoretic companion is alternating parity tree automata (APTA). It has been proved by Ong in 2006 [9] that MSO formulas can be checked automatically over the infinite trees generated by HORS, and that the decision procedure is n-EXPTIME complete, where n is the *order* of the HORS representing the infinite tree.

Several alternative proofs of Ong's result have been obtained by various teams around the world. It is worth mentionning the approach of Kobayashi and Ong [7], which relies on an intersection type discipline, and led to an analysis of HOMC at the light of linear logic by Grellois and Melliès [4, 5, 6, 3]. On this line of work, let us also mention Melliès' higher-order parity automata [8].

In recent work [2], Clairambault, Grellois and Murawski refined the complexity analysis of HOMC at the light of linear logic. They introduced *linear* HORS and *linear-nonlinear* APTA, and showed that the complexity of the HOMC problem depends in fact on the notion of *linear order*. The relation between linear and non-linear order is unclear: given an infinite tree generated by a linear HORS of linear order n, we do not know if it is possible to precisely bound the order m required to generate it using a (nonlinear) HORS of order m.

Internship Objectives:

HOMC is a challenging subject, which combines rewriting theory, automata theory, logical specifications, type theory, semantics (of linear logic)... The first purpose of the internship is to discover the subject and learn about the numerous objects that are required to work on it. As such, this internship would particularly fit someone willing to discover HOMC with the aim of potentially working further on it during his studies.

The second purpose would be to extend the result of [2] to the *selection property*. In the nonlinear setting, the selection property says that, when an infinite tree generated by a HORS is accepted by an APTA, one can compute a witness of this fact in n-EXPTIME, where n is the order of the scheme [1]. We conjecture that this property can also be lifted to the linear case, so that for linear HORS and linear-nonlinear APTA the complexity of the selection property would be m-EXPTIME, for m the *linear* order of the linear HORS of interest.

If time allows, and depending on the student's interests, many other research directions are possible. For instance, we could investigate the relation between linear and nonlinear order, investigate the relation of linear-nonlinear APTA with fragments of MSO,...

Administrative Information

- *Supervisor* : Charles Grellois, Aix-Marseille Université, Laboratoire LSIS-LIS, équipe LIRICA
- *Contact* : charles.grellois@univ-amu.fr
- *Internship Location*: Laboratoire LSIS (LIS from Jan 1st, 2018), Marseille

References

[1] Arnaud Carayol and Olivier Serre. Collapsible pushdown automata and labeled recursion schemes: Equivalence, safety and effective selection. In *LICS '12*, pages 165–174. IEEE Computer Society, 2012.

[2] Pierre Clairambault, Charles Grellois, and Andrzej Murawski. Linearity in higher-order recursion schemes, 2018.

[3] Charles Grellois. *Semantics of linear logic and higher-order model-checking. (Sémantique de la logique linéaire et "model-checking" d'ordre supérieur)*. PhD thesis, Paris Diderot University, France, 2016.

[4] Charles Grellois and Paul-André Melliès. Finitary semantics of linear logic and higher-order model-checking. In Giuseppe F. Italiano, Giovanni Pighizzini, and Donald Sannella, editors, *MFCS '15*, volume 9234 of *LNCS*, pages 256–268. Springer, 2015.

[5] Charles Grellois and Paul-André Melliès. An infinitary model of linear logic. In Andrew M. Pitts, editor, *FoSSaCS '15*, volume 9034 of *LNCS*, pages 41–55. Springer, 2015.

[6] Charles Grellois and Paul-André Melliès. Relational semantics of linear logic and higher-order model checking. In Stephan Kreutzer, editor, *CSL '15*, volume 41 of *LIPIcs*, pages 260–276. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2015.

[7] Naoki Kobayashi and C.-H. Luke Ong. A type system equivalent to the modal mu-calculus model checking of higher-order recursion schemes. In *LICS '09*, 2009.

[8] Paul-André Melliès. Higher-order parity automata. In *32nd Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2017, Reykjavik, Iceland, June 20-23, 2017*, pages 1–12. IEEE Computer Society, 2017.

[9] C.-H. Luke Ong. On model-checking trees generated by higher-order recursion schemes. In *LICS '06*, pages 81–90. IEEE Computer Society, 2006.