# Introduction to higher-order model-checking

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# What is model-checking?

# The halting problem

A natural question: does a program always terminate?

Undecidable problem (Turing 1936): a machine can not always determine the answer.

What if we use approximations?

# Model-checking

Approximate the program $\longrightarrow$ build a model $\mathcal{M}$.

Then, formulate a logical specification $\varphi$ over the model.

Aim: design a program which checks whether

$$\mathcal{M} \models \varphi.$$

That is, whether the model $\mathcal{M}$ meets the specification $\varphi$.

# An example

$$
\begin{aligned}
\texttt{Main} &= \texttt{Listen Nil} \\
\texttt{Listen } x &= \texttt{if end\_signal() then } x \\
&\quad\ \texttt{else Listen received\_data()} :: x
\end{aligned}
$$

# An example

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\begin{array}{rcl}
\texttt{Main} & = & \texttt{Listen Nil} \\
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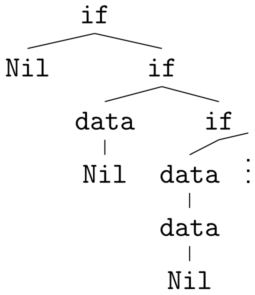
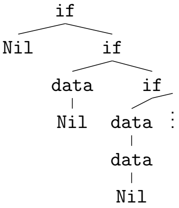
 & & \texttt{else Listen received\_data()} :: x
\end{array}
$$

A tree model:

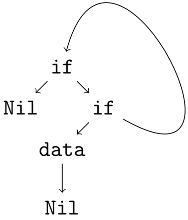



We abstracted conditionals and datatypes.
The approximation contains a non-terminating branch.

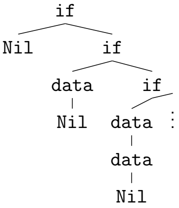
# Finite representations of infinite trees




is not regular: it is not the unfolding of a finite graph as



# Finite representations of infinite trees




but it is represented by a higher-order recursion scheme (HORS).

# Higher-order recursion schemes

Some regularity for infinite trees

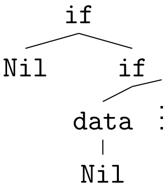
# Higher-order recursion schemes

$$\begin{array}{rcl}
\texttt{Main} & = & \texttt{Listen Nil} \\
\texttt{Listen}\ x & = & \texttt{if end_signal() then}\ x \\
 & & \texttt{else Listen received_data()} :: x
\end{array}$$

is abstracted as

$$\mathcal{G} = \begin{cases} \texttt{S} & = \quad \texttt{L Nil} \\ \texttt{L}\ x & = \quad \texttt{if}\ x\ (\texttt{L}\ (\texttt{data}\ x\ )\ ) \end{cases}$$

which represents the higher-order tree of actions



# Higher-order recursion schemes

$$\mathcal{G} \;=\; \begin{cases} \mathtt{S} & = \;\; \mathtt{L}\ \mathtt{Nil} \\ \mathtt{L}\ x & = \;\; \mathtt{if}\ x\,(\mathtt{L}\ (\mathtt{data}\ x\,)\,) \end{cases}$$

Rewriting starts from the start symbol $\mathtt{S}$:

$$\mathtt{S} \quad \to_{\mathcal{G}} \quad \begin{array}{c} \mathtt{L} \\ | \\ \mathtt{Nil} \end{array}$$

# Higher-order recursion schemes

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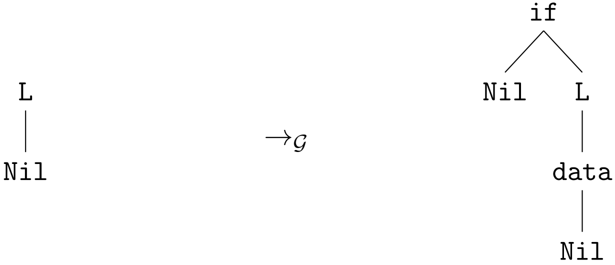

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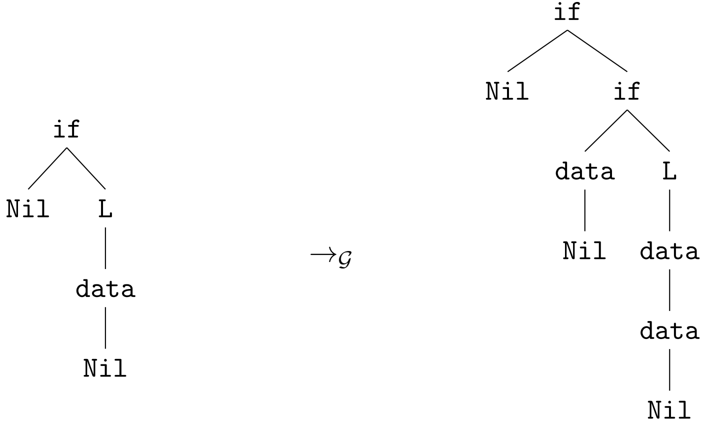
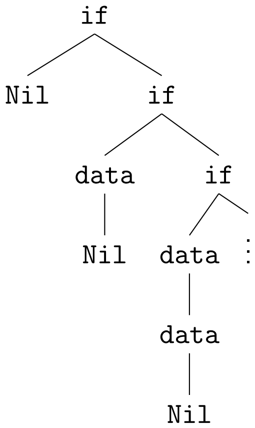


# Higher-order recursion schemes

$$\mathcal{G} = \begin{cases} \mathtt{S} & = \quad \mathtt{L\ Nil} \\ \mathtt{L}\ x & = \quad \mathtt{if}\ x\ (\mathtt{L}\ (\mathtt{data}\ x\ )\ ) \end{cases}$$

$$\langle \mathcal{G} \rangle \quad =$$



# Higher-order recursion schemes

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HORS can alternatively be seen as simply-typed $\lambda$-terms with

simply-typed recursion operators $Y_\sigma \ : \ (\sigma \rightarrow \sigma) \rightarrow \sigma$.

# Higher-order recursion schemes

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HORS can alternatively be seen as simply-typed $\lambda$-terms with

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# Alternating parity tree automata

Checking specifications over trees

(see Chapter 2)

# Monadic second order logic

MSO is a common logic in verification, allowing to express properties as:

« all executions halt »

« a given operation is executed infinitely often in some execution »

« every time data is added to a buffer, it is eventually processed »

# Alternating parity tree automata

Checking whether a formula holds can be performed using an automaton.

For an MSO formula $\varphi$, there exists an equivalent APT $\mathcal{A}_\varphi$ s.t.

$$\langle \mathcal{G} \rangle \;\vDash\; \varphi \quad \text{iff} \quad \mathcal{A}_\varphi \text{ has a run over } \langle \mathcal{G} \rangle.$$

APT = alternating tree automata (ATA) + parity condition.

# Alternating tree automata

ATA: non-deterministic tree automata whose transitions may duplicate or drop a subtree.

Typically: $\delta(q_0, \texttt{if}) \ = \ (2, q_0) \wedge (2, q_1)$.

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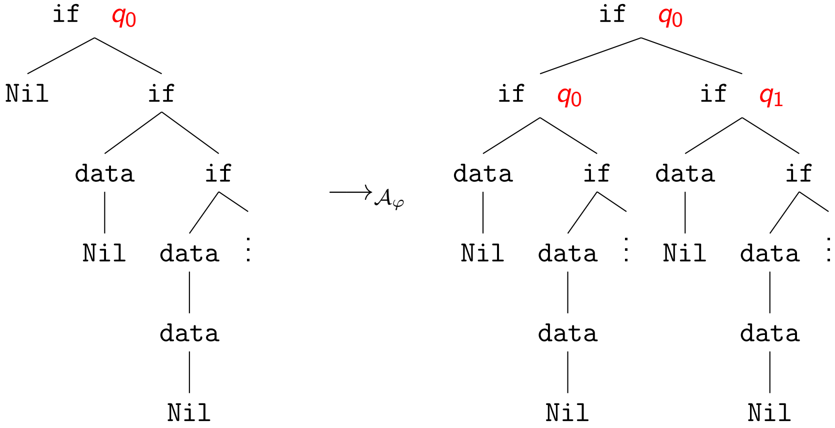
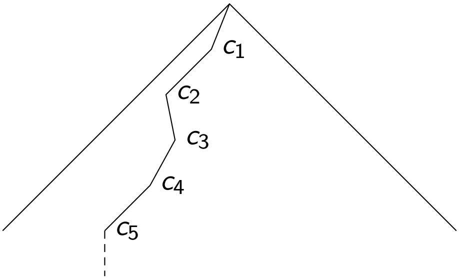


# Alternating parity tree automata

Each state of an APT is attributed a color

$$\Omega(q) \in Col \subseteq \mathbb{N}$$

An infinite branch of a run-tree is winning iff the maximal color among the ones occuring infinitely often along it is even.



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$$\Omega(q) \in Col \subseteq \mathbb{N}$$

An infinite branch of a run-tree is winning iff the maximal color among the ones occuring infinitely often along it is even.

A run-tree is winning iff all its infinite branches are.

For a MSO formula $\varphi$:

$$\mathcal{A}_\varphi \text{ has a winning run-tree over } \langle \mathcal{G} \rangle \qquad \text{iff} \qquad \langle \mathcal{G} \rangle \vDash \varphi.$$

# The higher-order model-checking problems

# The (local) HOMC problem

**Input:** HORS $\mathcal{G}$, formula $\varphi$.

**Output:** `true` if and only if $\langle \mathcal{G} \rangle \vDash \varphi$.

Example: $\varphi$ = « there is an infinite execution »




Output: `true`.

# The (local) HOMC problem

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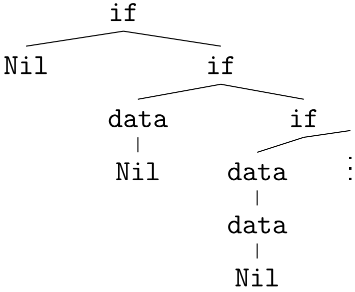



Output: `true`.

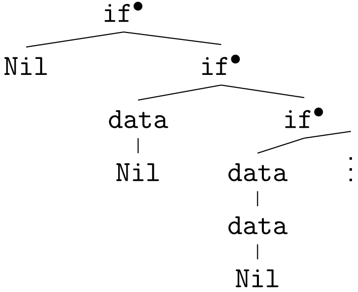
# The global HOMC problem

**Input:** HORS $\mathcal{G}$, formula $\varphi$.

**Output:** a HORS $\mathcal{G}^\bullet$ producing a marking of $\langle \mathcal{G} \rangle$.

Example: $\varphi \;=\;$ « there is an infinite execution »

Output: $\mathcal{G}^\bullet$ of value tree:



# The selection problem

**Input:** HORS $\mathcal{G}$, APT $\mathcal{A}$, state $q \in Q$.

**Output:** `false` if there is no winning run of $\mathcal{A}$ over $\langle \mathcal{G} \rangle$.
Else, a HORS $\mathcal{G}^q$ producing a such a winning run.

Example: $\varphi \ = \ $ « there is an infinite execution », $q_0$ corresponding to $\varphi$

Output: $\mathcal{G}^{q_0}$ producing

$$\begin{array}{c} \texttt{if}^{q_0} \\ | \\ \texttt{if}^{q_0} \\ | \\ \texttt{if}^{q_0} \\ | \\ \vdots \end{array}$$

# Purpose of my thesis

These three problems are decidable, with elaborate proofs (often) relying on semantics.

Our contribution: an excavation of the semantic roots of HOMC, at the light of linear logic, leading to refined and clarified proofs.

# Recognition by homomorphism

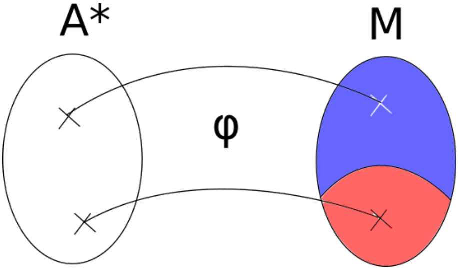
Where semantics comes into play

# Automata and recognition

For the usual finite automata on words: given a regular language $L \subseteq A^*$,

there exists a finite automaton $\mathcal{A}$ recognizing $L$

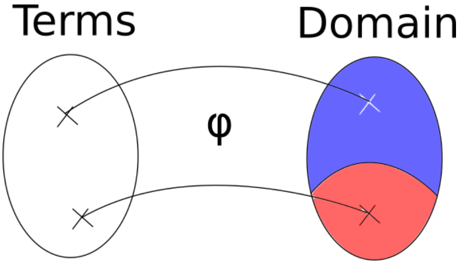
if and only if...




there exists a finite monoid $M$, a subset $K \subseteq M$
and a homomorphism $\varphi : A^* \to M$ such that $L = \varphi^{-1}(K)$.

# Automata and recognition

The picture we want:




(after Aehlig 2006, Salvati 2009)

but with recursion and w.r.t. an APT.

# Intersection types and alternation

A first connection with linear logic

# Alternating tree automata and intersection types

A key remark (Kobayashi 2009):

$$\delta(q_0, \texttt{if}) \;=\; (2, q_0) \wedge (2, q_1)$$

can be seen as the intersection typing

$$\texttt{if} \;:\; \emptyset \rightarrow (q_0 \wedge q_1) \rightarrow q_0$$

refining the simple typing

$$\texttt{if} \;:\; o \rightarrow o \rightarrow o$$

# Alternating tree automata and intersection types

In a derivation typing the tree `if` $T_1$ $T_2$ :

$$\dfrac{\text{App}\ \dfrac{\delta\ \dfrac{}{\emptyset \vdash \texttt{if} : \emptyset \to (q_0 \wedge q_1) \to q_0} \qquad \emptyset}{\emptyset \vdash \texttt{if}\ T_1 : (q_0 \wedge q_1) \to q_0} \qquad \dfrac{\vdots}{\emptyset \vdash T_2 : q_0} \qquad \dfrac{\vdots}{\emptyset \vdash T_2 : q_1}}{\emptyset \vdash \texttt{if}\ T_1\ T_2 : q_0}\ \text{App}$$

Intersection types naturally lift to higher-order – and thus to $\mathcal{G}$, which finitely represents $\langle \mathcal{G} \rangle$.

**Theorem (Kobayashi 2009)**

$\vdash \mathcal{G} : q_0$ *iff* *the ATA* $\mathcal{A}_\varphi$ *has a run-tree over* $\langle \mathcal{G} \rangle$.

# A closer look at the Application rule

In the intersection type system:

$$\text{App} \qquad \frac{\Delta \vdash t : (\ \theta_1\ \wedge \cdots \wedge\ \theta_n) \to \theta \qquad \Delta_i \vdash u\ :\ \theta_i}{\Delta\ ,\ \Delta_1\ ,\ \ldots\ ,\ \Delta_n\ \ \vdash\ \ t\,u\ :\ \theta}$$

This rule could be decomposed as:

$$\frac{\Delta\ \vdash\ t\ :\ (\ \bigwedge_{i=1}^{n}\ \theta_i\ ) \to \theta' \qquad \dfrac{\Delta_i\ \vdash\ u\ :\ \theta_i \qquad \forall i \in \{1, \ldots, n\}}{\Delta_1, \ldots, \Delta_n\ \vdash\ u\ :\ \bigwedge_{i=1}^{n}\ \theta_i}\ \text{Right}\ \bigwedge}{\Delta, \Delta_1, \ldots, \Delta_n\ \vdash\ t\ u\ :\ \theta'}$$

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# A closer look at the Application rule

$$\dfrac{\Delta \ \vdash \ t : (\bigwedge_{i=1}^{n} \ \theta_i) \rightarrow \theta' \qquad \dfrac{\Delta_i \ \vdash \ u : \theta_i \qquad \forall i \in \{1, \ldots, n\}}{\Delta_1, \ldots, \Delta_n \ \vdash \ u : \bigwedge_{i=1}^{n} \ \theta_i} \ \text{Right} \bigwedge}{\Delta, \Delta_1, \ldots, \Delta_n \ \vdash \ t \ u : \theta'}$$

Linear decomposition of the intuitionistic arrow:

$$A \Rightarrow B \quad = \quad ! \, A \multimap B$$

Two steps: duplication / erasure, then linear use.

Right $\bigwedge$ corresponds to the Promotion rule of indexed linear logic.
(see G.-Melliès, ITRS 2014)

# Intersection types and semantics of linear logic

$$A \Rightarrow B \;=\; !\,A \multimap B$$

Two interpretations of the exponential modality:

Qualitative models
(Scott semantics)

$$!\,A \;=\; \mathcal{P}_{fin}(A)$$

$$[\![o \Rightarrow o]\!] \;=\; \mathcal{P}_{fin}(Q) \times Q$$

$$\{q_0,\, q_0,\, q_1\} \;=\; \{q_0,\, q_1\}$$

Order closure

Quantitative models
(Relational semantics)

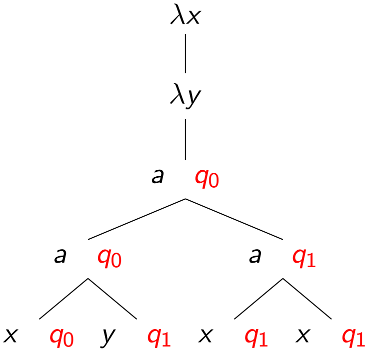
$$!\,A \;=\; \mathcal{M}_{fin}(A)$$

$$[\![o \Rightarrow o]\!] \;=\; \mathcal{M}_{fin}(Q) \times Q$$

$$[q_0,\, q_0,\, q_1] \;\neq\; [q_0,\, q_1]$$

Unbounded multiplicities

# An example of interpretation




In *Rel*, one denotation:

$$([q_0, q_1, q_1],\ [q_1],\ q_0)$$

In *ScottL*, a set containing the principal type

$$(\{q_0, q_1\},\ \{q_1\},\ q_0)$$

but also

$$(\{q_0, q_1, q_2\},\ \{q_1\},\ q_0)$$

and

$$(\{q_0, q_1\},\ \{q_0, q_1\},\ q_0)$$

and ...

# Intersection types and semantics of linear logic

$$
\begin{tikzcd}
Rel_! \arrow[r, leftrightarrow, "Bucciarelli-Ehrhard", "de\ \ Carvalho"'] \arrow[d, "Ehrhard"'] & \text{Non-idempotent types} \arrow[d, "Ehrhard"] \\
ScottL_! \arrow[r, leftrightarrow, "Ehrhard", "Terui"'] & \text{Idempotent types}
\end{tikzcd}
$$

(Bucciarelli-Ehrhard 2001, de Carvalho 2009, Ehrhard 2012, Terui 2012)

Fundamental idea:

$$[\![ t ]\!] \quad \cong \quad \{\, \theta \mid \emptyset \vdash \ t \ : \ \theta \,\}$$

for a closed term.

# Intersection types and semantics of linear logic

$$\begin{tikzcd}
Rel_! \arrow[rr, leftrightarrow, "Bucciarelli-Ehrhard", "de\ \ Carvalho"'] \arrow[d, "Ehrhard"'] & & \text{Non-idempotent types} \arrow[d, "Ehrhard"] \\
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\end{tikzcd}$$

Let $t$ be a term normalizing to a tree $\langle t \rangle$ and $\mathcal{A}$ be an alternating automaton.

$$\mathcal{A} \text{ accepts } \langle t \rangle \text{ from } q \quad \Leftrightarrow \quad q \in [\![ t ]\!] \quad \Leftrightarrow \quad \emptyset \vdash t : q :: o$$

(see Chapter 5)

Extension with recursion and parity condition?

# Adding parity conditions to the type system

# Alternating parity tree automata

We add coloring annotations to intersection types:

$$\delta(q_0, \texttt{if}) \;=\; (2, q_0) \wedge (2, q_1)$$

now corresponds to

$$\texttt{if} \;:\; \emptyset \to \left( \Box_{\Omega(q_0)}\, q_0 \wedge \Box_{\Omega(q_1)}\, q_1 \right) \to q_0$$

Idea: `if` is a run-tree with two holes:

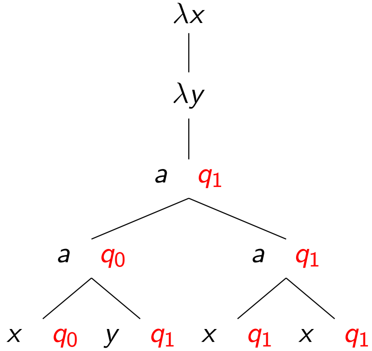
$$\begin{array}{c} \texttt{if} \\ [\,]_{q_0} \quad [\,]_{q_1} \end{array}$$

A new neutral (least) color: $\epsilon$.

We refine the approach of Kobayashi and Ong in a modal way (see Chapter 6).

# An example of colored intersection type

Set $\Omega(q_0) \ = \ 0$ and $\Omega(q_1) \ = \ 1$.




has now type

$$\Box_0\, q_0 \wedge \Box_1\, q_1 \rightarrow \Box_1\, q_1 \rightarrow q_1$$

Note the color 0 on $q_0$...

# A type-system for verification (Grellois-Melliès 2014)

Axiom $\dfrac{}{x \,:\, \Box_{\epsilon}\ \theta_i \ \vdash\ x \,:\, \theta_i}$

$$\delta \qquad \frac{\{\,(i, q_{ij}) \mid 1 \leq i \leq n, 1 \leq j \leq k_i\,\} \quad \text{satisfies} \quad \delta_A(q, a)}{\emptyset \vdash a \,:\, \bigwedge_{j=1}^{k_1} \Box_{\Omega(q_{1j})}\ q_{1j} \ \rightarrow\ \ldots\ \rightarrow\ \bigwedge_{j=1}^{k_n} \Box_{\Omega(q_{nj})}\ q_{nj} \rightarrow q}$$

$$\text{App} \qquad \frac{\Delta \vdash t : (\Box_{m_1}\ \theta_1\ \wedge \cdots \wedge \Box_{m_k}\ \theta_k) \rightarrow \theta \qquad \Delta_i \vdash u \,:\, \theta_i}{\Delta\ +\ \Box_{m_1}\Delta_1\ +\ \ldots\ +\ \Box_{m_k}\Delta_k\ \vdash\ t\ u \,:\, \theta}$$

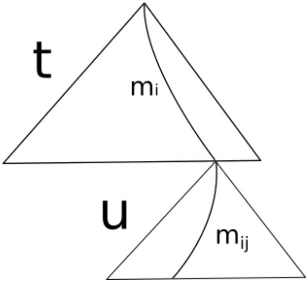
$$\lambda \qquad \frac{\Delta\,,\, x \,:\, \bigwedge_{i \in I} \Box_{m_i}\ \theta_i \ \vdash\ t \,:\, \theta}{\Delta\ \vdash\ \lambda\, x\,.\, t \,:\, \left(\bigwedge_{i \in I} \Box_{m_i}\ \theta_i\right) \rightarrow \theta}$$

$$\mathit{fix} \qquad \frac{\Gamma \vdash \mathcal{R}(F) \,:\, \theta}{F \,:\, \Box_{\epsilon}\ \theta\ \vdash\ F \,:\, \theta}$$

# A type-system for verification

A colored Application rule:

$$\text{App} \quad \frac{\Delta \vdash t : (\square_{m_1}\ \theta_1\ \wedge \cdots \wedge \square_{m_k}\ \theta_k) \to \theta \qquad \Delta_i \vdash u\ :\ \theta_i}{\Delta\ +\ \square_{m_1}\Delta_1\ +\ \ldots\ +\ \square_{m_k}\Delta_k\ \vdash\ t\ u\ :\ \theta}$$



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inducing a winning condition on infinite proofs: the node

$$\Delta_i \vdash u\ :\ \theta_i$$

has color $m_i$, others have color $\epsilon$, and we use the parity condition.

# A type-system for verification

We now capture all MSO (see Chapter 6-8):

**Theorem (G.-Melliès 2014, from Kobayashi-Ong 2009)**

*$S : q_0 \vdash S : q_0$ admits a winning typing derivation iff the alternating parity automaton $\mathcal{A}$ has a winning run-tree over $\langle \mathcal{G} \rangle$.*

We obtain decidability by considering idempotent types.

Our reformulation

- shows the modal nature of $\Box$ (in the sense of S4),
- internalizes the parity condition,
- paves the way for semantic constructions.

# Colored models of linear logic

# A closer look at the Application rule

$$\frac{\Delta \vdash t : (\square_{m_1}\ \theta_1\ \wedge \cdots \wedge \square_{m_k}\ \theta_k) \rightarrow \theta \qquad \Delta_i \vdash u\ :\ \theta_i}{\Delta\ +\ \square_{m_1}\Delta_1\ +\ \ldots\ +\ \square_{m_k}\Delta_k\ \ \vdash\ \ t\, u\ :\ \theta}$$

could be decomposed as:

$$\dfrac{\Delta \vdash t : \left(\bigwedge_{i=1}^{k}\ \square_{m_i}\ \theta_i\right) \rightarrow \theta \qquad \dfrac{\dfrac{\Delta_1 \vdash u\ :\ \theta_1}{\square_{m_1}\ \Delta_1\ \vdash\ u\ :\ \square_{m_1}\ \theta_1} \quad \ldots \quad \dfrac{\Delta_k \vdash u\ :\ \theta_k}{\square_{m_k}\ \Delta_k\ \vdash\ u\ :\ \square_{m_k}\ \theta_k}\ \text{Right}\ \square}{\square_{m_1}\Delta_1,\ \ldots,\ \square_{m_k}\Delta_k\ \vdash\ u\ :\ \bigwedge_{i=1}^{k}\ \square_{m_i}\ \theta_i}\ \text{Right}\ \bigwedge}{\Delta,\ \square_{m_1}\Delta_1,\ \ldots,\ \square_{m_k}\Delta_k\ \ \vdash\ \ t\, u\ :\ \theta}$$

Right $\square$ looks like a promotion. In linear logic:

$$A \Rightarrow B\ \ =\ \ !\,\square\, A \multimap B$$

We show that the modality $\square$ distributes over the exponential in the semantics.

# Colored semantics

We extend:

- *Rel* with countable multiplicities, coloring and an inductive-coinductive fixpoint (Chapter 9)
- *ScottL* with coloring and an inductive-coinductive fixpoint (Chapter 10).

Methodology: think in the relational semantics, and adapt to the Scott semantics using Ehrhard's 2012 result:

the finitary model *ScottL* is the extensional collapse of *Rel*.

# Infinitary relational semantics

Extension of *Rel* with infinite multiplicities:

$$\lightning A \;\; = \;\; \mathcal{M}_{count}(A)$$

and coloring modality (parametric comonad)

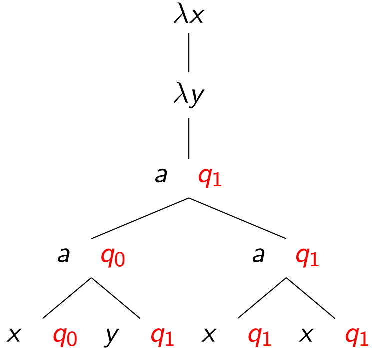
$$\Box A \;\; = \;\; Col \times A$$

Composite comonad: $\boldsymbol{\lightning} \;\; = \;\; \lightning \, \Box$ is an exponential.

Induces a colored CCC $Rel_{\lightning}$ ($\rightarrow$ model of the $\lambda$-calculus).

# An example of interpretation

Set $\Omega(q_i) \ = \ i$.




has denotation

$$([(0, q_0), (1, q_1), (1, q_1)], [(1, q_1)], q_1)$$

(corresponding to the type $\Box_0 \, q_0 \wedge \Box_1 \, q_1 \to \Box_1 \, q_1 \to q_1$)

# Model-checking and infinitary semantics

Inductive-coinductive fixpoint operator: composes denotations w.r.t. the parity condition.

**Theorem**

*An APT $\mathcal{A}$ has a winning run from $q_0$ over $\langle \mathcal{G} \rangle$ if and only if*

$$q_0 \in [\![ \lambda(\mathcal{G}) ]\!]_{\mathcal{A}}$$

*where $\lambda(\mathcal{G})$ is a $\lambda Y$-term corresponding to $\mathcal{G}$.*

**Conjecture**

*An APT $\mathcal{A}$ has a winning run from $q_0$ over $\langle \mathcal{G} \rangle$ if and only if*

$$q_0 \in [\![ \lambda(\mathcal{G})^{\Sigma} ]\!] \circ [\![ \delta^{\dagger} ]\!]$$

*where $\lambda(\mathcal{G})^{\Sigma}$ is a Church encoding of a $\lambda Y$-term corresponding to $\mathcal{G}$.*

# Finitary semantics

In ScottL, we define $\Box$, $\lambda$ and $\mathbf{Y}$ similarly (using downward-closures).
*ScottL*$_{\lightning}$ is a model of the $\lambda Y$-calculus.

**Theorem**

*An APT $\mathcal{A}$ has a winning run from $q_0$ over $\langle \mathcal{G} \rangle$ if and only if*

$$q_0 \in [\![ \lambda(\mathcal{G}) ]\!].$$

**Corollary**

*The local higher-order model-checking problem is decidable (and is n-EXPTIME complete).*

**Theorem**

*The selection problem is decidable.*

# Perspectives

- A purely coinductive proof of the soundness-and-completeness theorem
- Accommodating the modal approach to other classes of automata
- Understanding the infinitary semantics
- Logical aspects: colored tensorial logic, fixpoints. . .
- Game semantics interpretations?
- Is the complexity related to light linear logics?
- Extensional collapse between the two colored models?