Systèmes de preuve pour les logiques de "Bringing-it-About"

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Outline

1. BIAT logics in a nutshell
2. Semantics
3. Proof systems

Logics of bringing-it-about

Two main routes in agency logics (cf. Herzig, Lorini & Troquard 2018)

1. Actions as results
   - Analysis of actions only in terms of their result.
   - e.g. logics of Bringing-It-About-That, logics of Seeing-To-It-That.

2. Actions as means+results
   - Focus on the result and the means by which it is obtained.
   - e.g. variants of Propositional Dynamic Logic.

$\neq$

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   - e.g. variants of Propositional Dynamic Logic.

Logic of bringing-it-about

> action = the result it brings about

No matter the means by which the result is obtained.

An agent b.i.a.t. something only if she is responsible of its realization

- Cannot b.i.a.t. something which is the case independently from her action.

- Delegate $\neq$ Do.

Elgesem logic: BIAT + capability (Elgesem 1997)

Two modalities: Does & Can, indexed by agents

- $\mathbb{E}_i A$ "Agent i b.i.a.t. A".
- $\mathbb{C}_i A$ "Agent i is capable of b.i.a.t. A".

E.g.

$$\mathbb{C}_{Sara} \neg \mathbb{E}_{Lucy} BankTransfer$$

"Sara can prevent Lucy from making a bank transfer".

Elgesem logic

- Classical propositional logic +
- Principles for $\mathbb{E}$ and $\mathbb{C}$.

Elgesem logic: BIAT + capability (Elgesem 1997)

- Principle of success:

$$(\mathrm{T}_{\mathbb{E}}) \quad \mathbb{E}_i A \rightarrow A$$

- Principle of aggregation:

$$(\mathrm{C}_{\mathbb{E}}) \quad \mathbb{E}_i A \wedge \mathbb{E}_i B \rightarrow \mathbb{E}_i (A \wedge B)$$

- Do implies Can:

$$(\mathrm{Int}_{\mathbb{E}\mathbb{C}}) \quad \mathbb{E}_i A \rightarrow \mathbb{C}_i A$$

- Principle of possibility:

$$(\mathrm{P}_{\mathbb{C}}) \quad \neg \mathbb{C}_i \bot$$

- Principle of avoidability:

$$(\mathrm{Q}_{\mathbb{C}}) \quad \neg \mathbb{C}_i \top$$

- Actions are not sensitive to their syntactic formulation:

$$(\mathrm{RE}_{\mathbb{E}}) \quad \frac{A \leftrightarrow B}{\mathbb{E}_i A \leftrightarrow \mathbb{E}_i B} \qquad (\mathrm{RE}_{\mathbb{C}}) \quad \frac{A \leftrightarrow B}{\mathbb{C}_i A \leftrightarrow \mathbb{C}_i B}$$

Remark: P, Q for both $\mathbb{C}$ and $\mathbb{E}$. T, C only for $\mathbb{E}$.

VERY NORMAL PEOPLE

VERY NON-NORMAL LOGIC

Non-normal modalities

- No monotonicity:

$$\models A \rightarrow B \;\not\Rightarrow\; \models \mathbb{E}_i A \rightarrow \mathbb{E}_i B$$

(otherwise $\mathbb{E}_i A \rightarrow \mathbb{E}_i \top$)

- No necessitation:

$$\models A \;\not\Rightarrow\; \models \mathbb{E}_i A$$

(otherwise $\mathbb{E}_i \top$)

Incompatible with normal modalities

- Contains the negation of necessitation:

$$\models \neg \mathbb{E}_i \top$$

- No normal extension is possible

A basic framework that can be extended in many ways

Some examples:

- Attempted actions (Jones & Parent 2007):

  $\mathbb{H}_i A$ "Agent i attempts to b.i.a.t. A".

- Time, confirmation and disconfirmation (Troquard 2019):

  $(\mathbb{C}_i A)\mathcal{S}(\mathbb{E}_j B)$ "Agent i could do A since agent j did B".

- Coalitions (Troquard 2014):

  $\mathbb{E}_g A$ "Group g b.i.a.t. A".

A basic framework that can be extended in many ways

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We look at this one

No coalition monotonicity

A group b.i.a.t. something only if every member contributes to its realization

Troquard's coalition logic (Troquard 2014)

- Elgesem axioms with single agents replaced by groups:

$$\mathbb{E}_g A \to A \qquad \mathbb{E}_g A \wedge \mathbb{E}_g B \to \mathbb{E}_g (A \wedge B) \qquad \neg \mathbb{C}_g \bot \qquad \neg \mathbb{C}_g \top$$

$$\mathbb{E}_g A \to \mathbb{C}_g A \qquad \frac{A \leftrightarrow B}{\mathbb{E}_g A \leftrightarrow \mathbb{E}_g B} \qquad \frac{A \leftrightarrow B}{\mathbb{C}_g A \leftrightarrow \mathbb{C}_g B}$$

- Principle of non-emptiness:

$$(\mathsf{F}_{\mathbb{C}}) \quad \neg \mathbb{C}_{\emptyset} A$$

- Principle of coalition:

$$(\mathsf{Int}^2_{\mathbb{E}\mathbb{C}}) \quad \mathbb{E}_{g_1} A \wedge \mathbb{E}_{g_2} B \to \mathbb{C}_{g_1 \cup g_2} (A \wedge B)$$

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2. Semantics
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Semantics of Elgesem logic

- Selection function models (Elgesem 1997)
- Neighbourhood models (Governatori & Rotolo 2005)
- Bi-neighbourhood models 

We look at these two

Neighbourhood models

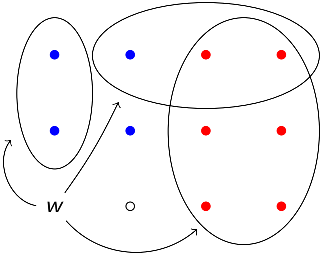
$\mathcal{M} = \langle \mathcal{W}, \mathcal{N}_i^{\mathbb{E}}, \mathcal{N}_i^{\mathbb{C}}, \mathcal{V} \rangle$, where

- $\mathcal{W}$ non-empty set of worlds.
- $\mathcal{V}$ valuation function $Atm \longrightarrow \mathcal{P}(W)$.
- $\mathcal{N}_i^{\mathbb{E}}$, $\mathcal{N}_i^{\mathbb{C}}$ neighbourhood functions $\mathcal{W} \longrightarrow \mathcal{PP}(W)$ for every agent i.

Intuition: $\mathcal{N}_i^{\mathbb{E}}, \mathcal{N}_i^{\mathbb{C}}$ assign to every world the actions that i does/can do in it

$$w \Vdash \mathbb{E}_i A \quad \text{iff} \quad [\![A]\!] \in \mathcal{N}_i^{\mathbb{E}}(w) \qquad\qquad w \Vdash \mathbb{C}_i A \quad \text{iff} \quad [\![A]\!] \in \mathcal{N}_i^{\mathbb{C}}(w)$$

• p
• q



$w \Vdash \mathbb{E}_i p$
$w \not\Vdash \mathbb{E}_i q$

Neighbourhood semantics (Governatori & Rotolo 2005)

Neighbourhood models

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$$w \Vdash \mathbb{E}_i A \quad \text{iff} \quad [\![A]\!] \in \mathcal{N}_i^{\mathbb{E}}(w) \qquad\qquad w \Vdash \mathbb{C}_i A \quad \text{iff} \quad [\![A]\!] \in \mathcal{N}_i^{\mathbb{C}}(w)$$

Model conditions

$$\text{If } \alpha, \beta \in \mathcal{N}_i^{\mathbb{E}}(w), \text{ then } \alpha \cap \beta \in \mathcal{N}_i^{\mathbb{E}}(w). \qquad (\mathrm{C}_{\mathbb{E}})$$

$$\text{If } \alpha \in \mathcal{N}_i^{\mathbb{E}}(w), \text{ then } w \in \alpha. \qquad (\mathrm{T}_{\mathbb{E}})$$

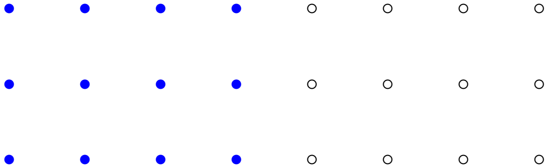
$$\emptyset \notin \mathcal{N}_i^{\mathbb{C}}(w). \qquad (\mathrm{P}_{\mathbb{C}})$$

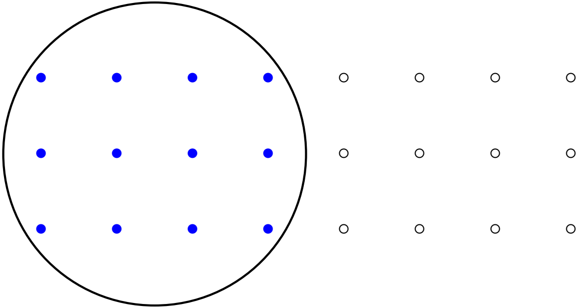
$$\mathcal{W} \notin \mathcal{N}_i^{\mathbb{C}}(w). \qquad (\mathrm{Q}_{\mathbb{C}})$$

$$\mathcal{N}_i^{\mathbb{E}}(w) \subseteq \mathcal{N}_i^{\mathbb{C}}(w). \qquad (\mathrm{Int}_{\mathbb{EC}})$$

Bi-neighbourhood semantics: Reasoning with partial information

● p ○ $\neg p$



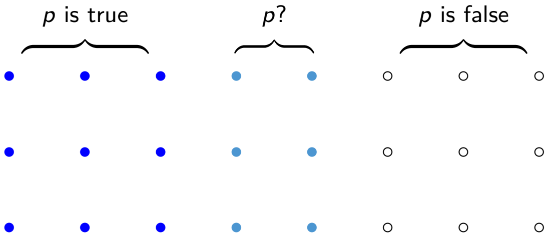


$\in \mathcal{N}_i^{\mathbb{E}}(w)$

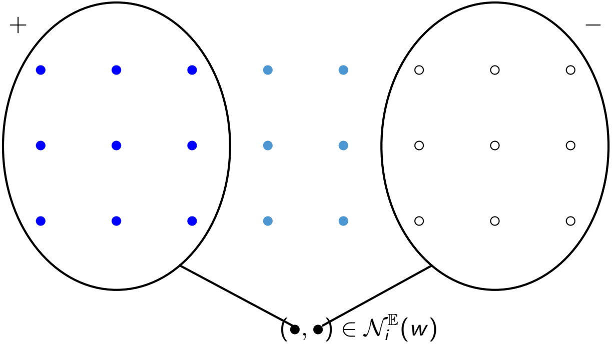
$\Downarrow$

$w \Vdash \mathbb{E}_i p$

Bi-neighbourhood semantics: Reasoning with partial information



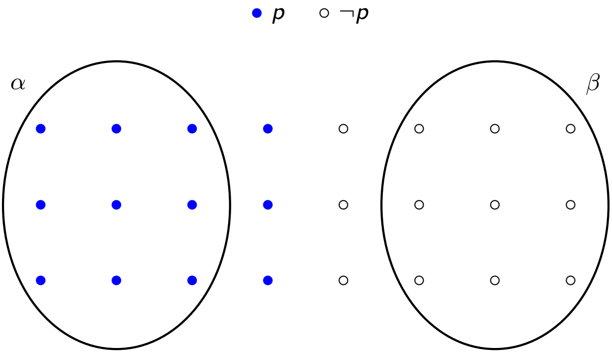
Bi-neighbourhood semantics: Reasoning with partial information



$$\Downarrow$$

$$w \Vdash \mathbb{E}_i p$$

Bi-neighbourhood semantics: Reasoning with partial information



$w \Vdash \mathbb{E}_i p$ iff there is $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{E}}(w)$ s.t. $\alpha \subseteq [\![p]\!]$ and $\beta \subseteq [\![\neg p]\!]$.

Bi-neighbourhood semantics

$\mathcal{M} = \langle \mathcal{W}, \mathcal{N}, \mathcal{V} \rangle$, where $\mathcal{W} \neq \emptyset$; $\mathcal{V} : Atm \longrightarrow \mathcal{P}(W)$; and

- $\mathcal{N}_i^{\mathbb{E}}$, $\mathcal{N}_i^{\mathbb{C}}$ bi-neighbourhood functions $\mathcal{W} \longrightarrow \mathcal{P}(\mathcal{P}(W) \times \mathcal{P}(W))$.

$w \Vdash \mathbb{E}_i A$ iff there is $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{E}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.

$w \Vdash \mathbb{C}_i A$ iff there is $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{C}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.

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$w \Vdash \mathbb{C}_i A$	iff	there is $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{C}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.

Model conditions

If $(\alpha, \beta), (\gamma, \delta) \in \mathcal{N}_i^{\mathbb{E}}(w)$, then $(\alpha \cap \gamma, \beta \cup \delta) \in \mathcal{N}_i^{\mathbb{E}}(w)$.	$(C_{\mathbb{E}})$
If $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{E}}(w)$, then $w \in \alpha$.	$(T_{\mathbb{E}})$
If $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{C}}(w)$, then $\beta \neq \emptyset$.	$(Q_{\mathbb{C}})$
If $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{C}}(w)$, then $\alpha \neq \emptyset$.	$(P_{\mathbb{C}})$
$\mathcal{N}_i^{\mathbb{E}}(w) \subseteq \mathcal{N}_i^{\mathbb{C}}(w)$.	$(\mathrm{Int}_{\mathbb{E}\mathbb{C}})$

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$w \Vdash \mathbb{C}_i A$	iff	there is $(\alpha, \beta) \in \mathcal{N}_i^{\mathbb{C}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.

Relation with the neighbourhood semantics

- (α, β) can be seen as lower and upper bounds of standard neighbourhoods: Equivalent standard models definable with:

$$_{(st)}\mathcal{N}_i^{\mathbb{E}}(w) = \{\gamma \mid \text{there is } (\alpha, \beta) \in {}_{(bi)}\mathcal{N}_i^{\mathbb{E}}(w) \text{ s.t. } \alpha \subseteq \gamma \subseteq \mathcal{W} \setminus \beta\}.$$

- Neighbourhood semantics as the particular case where α and β are complementary for every (α, β).

Semantics for coalition logic

- Neighbourhood semantics (Troquard 2014)

Bi-neighbourhood semantics 

- $\mathcal{N}_g^{\mathbb{E}}$, $\mathcal{N}_g^{\mathbb{C}}$ bi-neighbourhood functions $\mathcal{W} \longrightarrow \mathcal{P}(\mathcal{P}(W) \times \mathcal{P}(W))$ for every group g.
- $w \Vdash \mathbb{E}_g A$ iff there is $(\alpha, \beta) \in \mathcal{N}_g^{\mathbb{E}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.
- $w \Vdash \mathbb{C}_g A$ iff there is $(\alpha, \beta) \in \mathcal{N}_g^{\mathbb{C}}(w)$ s.t. $\alpha \subseteq [\![A]\!]$ and $\beta \subseteq [\![\neg A]\!]$.
- Model conditions:

If $(\alpha, \beta), (\gamma, \delta) \in \mathcal{N}_i^{\mathbb{E}}(w)$, then $(\alpha \cap \gamma, \beta \cup \delta) \in \mathcal{N}_i^{\mathbb{E}}(w)$. ($C_{\mathbb{E}}$)

If $(\alpha, \beta) \in \mathcal{N}_g^{\mathbb{E}}(w)$, then $w \in \alpha$. ($T_{\mathbb{E}}$)

If $(\alpha, \beta) \in \mathcal{N}_g^{\mathbb{C}}(w)$, then $\beta \neq \emptyset$. ($Q_{\mathbb{C}}$)

If $(\alpha, \beta) \in \mathcal{N}_g^{\mathbb{C}}(w)$, then $\alpha \neq \emptyset$. ($P_{\mathbb{C}}$)

$\mathcal{N}_g^{\mathbb{E}}(w) \subseteq \mathcal{N}_g^{\mathbb{C}}(w)$. ($\mathrm{Int}_{\mathbb{E}\mathbb{C}}$)

$\mathcal{N}_\emptyset^{\mathbb{C}}(w) = \emptyset$. ($F_{\mathbb{C}}$)

If $(\alpha, \beta) \in \mathcal{N}_{g_1}^{\mathbb{E}}(w)$ and $(\gamma, \delta) \in \mathcal{N}_{g_2}^{\mathbb{E}}(w)$, then $(\alpha \cap \gamma, \beta \cup \delta) \in \mathcal{N}_{g_1 \cup g_2}^{\mathbb{C}}(w)$. ($\mathrm{Int}^2_{\mathbb{E}\mathbb{C}}$)

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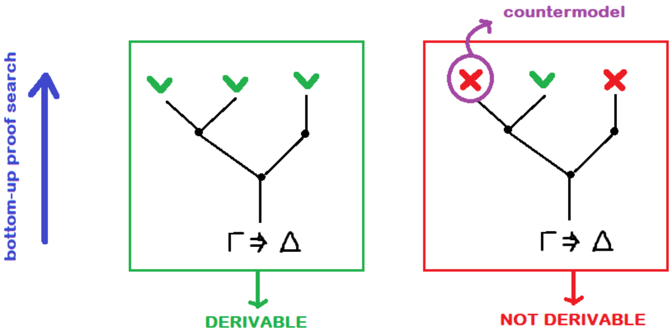
Desiderata on the proof system

1. Terminating proof search procedure.
2. Countermodel extraction from every single failed proof.



⇒ Constructive decision procedure: for every formula a proof or a countermodel.

Hypersequents with blocks

Sequent calculi extended with additional structural connectives

Block: $\langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Sigma \rangle_j^{\mathbb{C}}$, where Σ multiset of formulas.

Sequent: $\Gamma, \langle \Sigma_1 \rangle_i^{\mathbb{E}}, ..., \langle \Sigma_n \rangle_j^{\mathbb{C}} \Rightarrow \Delta$.

Hypersequent: $\Gamma_1 \Rightarrow \Delta_1 \mid ... \mid \Gamma_n \Rightarrow \Delta_n$.

Formula interpretation i

$$\begin{array}{lcl}
\langle A_1, ..., A_n \rangle_i^{\mathbb{E}} & \rightsquigarrow & \mathbb{E}_i(A_1 \wedge ... \wedge A_n) \\
\langle A_1, ..., A_n \rangle_j^{\mathbb{C}} & \rightsquigarrow & \mathbb{C}_j(A_1 \wedge ... \wedge A_n) \\
\Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, ..., \langle \Pi \rangle_j^{\mathbb{C}} \Rightarrow \Delta & \rightsquigarrow & \bigwedge \Gamma \wedge \mathbb{E}_i \bigwedge \Sigma \wedge ... \wedge \mathbb{C}_j \bigwedge \Pi \rightarrow \bigvee \Delta.
\end{array}$$

Semantic interpretation

$$\begin{array}{lll}
w \Vdash \Gamma \Rightarrow \Delta & \text{iff} & w \Vdash i(\Gamma \Rightarrow \Delta). \\
\mathcal{M} \models \Gamma \Rightarrow \Delta & \text{iff} & w \Vdash \Gamma \Rightarrow \Delta \text{ for every } w \text{ of } \mathcal{M}. \\
\mathcal{M} \models \Gamma_1 \Rightarrow \Delta_1 \mid ... \mid \Gamma_n \Rightarrow \Delta_n & \text{iff} & \mathcal{M} \models \Gamma_i \Rightarrow \Delta_i \text{ for some } i \in \{1, ..., n\}.
\end{array}$$

$$\mathsf{L}_{\mathbb{E}}\ \frac{G \mid \Gamma, \mathbb{E}_i A, \langle A \rangle_i^{\mathbb{E}} \Rightarrow \Delta}{G \mid \Gamma, \mathbb{E}_i A \Rightarrow \Delta} \qquad \mathsf{L}_{\mathbb{C}}\ \frac{G \mid \Gamma, \mathbb{C}_i A, \langle A \rangle_i^{\mathbb{C}} \Rightarrow \Delta}{G \mid \Gamma, \mathbb{C}_i A \Rightarrow \Delta}$$

$$\mathsf{R}_{\mathbb{E}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i A, \Delta \mid \Sigma \Rightarrow A \qquad \{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i A, \Delta \mid A \Rightarrow B\}_{B \in \Sigma}}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i A, \Delta}$$

$$\mathsf{R}_{\mathbb{C}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \mathbb{C}_i A, \Delta \mid \Sigma \Rightarrow A \qquad \{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \mathbb{C}_i A, \Delta \mid A \Rightarrow B\}_{B \in \Sigma}}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \mathbb{C}_i A, \Delta}$$

$$\mathsf{C}_{\mathbb{E}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Pi \rangle_i^{\mathbb{E}}, \langle \Sigma, \Pi \rangle_i^{\mathbb{E}} \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Pi \rangle_i^{\mathbb{E}} \Rightarrow \Delta} \qquad \mathsf{T}_{\mathbb{E}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \Sigma \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \Delta}$$

$$\mathsf{Q}_{\mathbb{C}}\ \frac{\{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta \mid \Rightarrow B\}_{B \in \Sigma}}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta} \qquad \mathsf{P}_{\mathbb{C}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta \mid \Sigma \Rightarrow}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta}$$

$$\mathsf{Int}_{\mathbb{EC}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \Delta}$$

- Separate left and right rules for $\mathbb{E}$ and $\mathbb{C}$.
- One rule for every characteristic axiom.
- Cumulative rules: principal formulas always copied into the premiss.

Semantic intuition

- Components represent the worlds of a model.
- Blocks represent truth sets of formulas: $\langle A \rangle_i^{\mathbb{E}} \approx [\![A]\!] \in \mathcal{N}_i^{\mathbb{E}}(w)$.
- Rules express axiom conditions in the neighbourhood semantics:

$$\mathsf{L}_{\mathbb{E}} \; \frac{G \mid \Gamma, \mathbb{E}_i A, \langle A \rangle_i^{\mathbb{E}} \Rightarrow \Delta}{G \mid \Gamma, \mathbb{E}_i A \Rightarrow \Delta} \qquad w \Vdash \mathbb{E}_i A \Longrightarrow [\![A]\!] \in \mathcal{N}_i^{\mathbb{E}}(w).$$

$$\mathsf{C}_{\mathbb{E}} \; \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Pi \rangle_i^{\mathbb{E}}, \langle \Sigma, \Pi \rangle_i^{\mathbb{E}} \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Pi \rangle_i^{\mathbb{E}} \Rightarrow \Delta} \qquad [\![\textstyle\bigwedge \Sigma]\!], [\![\textstyle\bigwedge \Pi]\!] \in \mathcal{N}_i^{\mathbb{E}}(w) \Longrightarrow [\![\textstyle\bigwedge \Sigma]\!] \cap [\![\textstyle\bigwedge \Pi]\!] \in \mathcal{N}_i^{\mathbb{E}}(w).$$

$$\mathsf{Int}_{\mathbb{E}\mathbb{C}} \; \frac{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}}, \langle \Sigma \rangle_i^{\mathbb{C}} \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle_i^{\mathbb{E}} \Rightarrow \Delta} \qquad [\![\textstyle\bigwedge \Sigma]\!] \in \mathcal{N}_i^{\mathbb{E}}(w) \Longrightarrow [\![\textstyle\bigwedge \Sigma]\!] \in \mathcal{N}_i^{\mathbb{C}}(w).$$

- Cumulative rules: A saturated hypersequent contains all information needed to build a countermodel.

Hypersequents and backtracking

Without hypersequents

non derivable (for $p \Rightarrow q$) — backtracking →

$$
\cfrac{\cfrac{\cfrac{p, q \Rightarrow p}{p \wedge q \Rightarrow p} \qquad \cfrac{p \Rightarrow p \qquad p \Rightarrow q}{p \Rightarrow p \wedge q}}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p)}\,\mathsf{R}_{\mathbb{E}}}{\mathbb{E}_i(p \wedge q) \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p)}\,\mathsf{L}_{\mathbb{E}}
$$

derivable / derivable

$$
\cfrac{\cfrac{\vdots}{p \wedge q \Rightarrow q \wedge p} \qquad \cfrac{\vdots}{q \wedge p \Rightarrow p \wedge q}}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p)}\,\mathsf{R}_{\mathbb{E}}
$$

With hypersequents

$$
\cfrac{\cfrac{\vdots \qquad \cfrac{\vdots \qquad \cfrac{\vdots \qquad \cfrac{\ldots \mid q, p \Rightarrow p \qquad \ldots \mid q, p \Rightarrow q}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p) \mid p \Rightarrow q \mid q \wedge p \Rightarrow p \wedge q}}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p) \mid p \Rightarrow q}\,\mathsf{R}_{\mathbb{E}}}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p) \mid p \Rightarrow p \wedge q}\,\mathsf{L}\wedge}{\langle p \wedge q \rangle_i^{\mathbb{E}} \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p)}\,\mathsf{R}_{\mathbb{E}}}{\mathbb{E}_i(p \wedge q) \Rightarrow \mathbb{E}_i p, \mathbb{E}_i(q \wedge p)}\,\mathsf{L}_{\mathbb{E}}
$$

$\Rightarrow$ Decision procedure by a single proof

Trade-off: Optimality vs. countermodels

Properties

- Admissibility of structural rules and syntactic cut elimination
- Termination of proof search (avoid redundant rule applications)

Sequents vs. hypersequents with blocks

- Sequents (Lellmann 2013)
  - No direct extraction of countermodels
  - PSPACE proof-search
- Hypersequents with blocks
  - Direct extraction of countermodels
  - Sub-optimal proof-search n $\mathbb{E}$-subformulas $\Rightarrow 2^n$ blocks

A necessary trade-off?

- A failed proof explicitly builds a model: A component for every world.
- Conjecture: Satisfiable formulas of size n whose smallest models have 2^n worlds. Known for **K** (Blackburn et al. 2001).

Countermodel extraction from saturated hypersequent

- Every component corresponds to a world.
- Formulas in Γ are true, formulas in Δ are false.

Countermodel extraction from saturated hypersequent

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- Formulas in Γ are true, formulas in Δ are false.

$$\Gamma_1 \Rightarrow \Delta_1 \mid \Gamma_2 \Rightarrow \Delta_2 \mid \Gamma_3 \Rightarrow \Delta_3 \mid \Gamma_4 \Rightarrow \Delta_4 \mid \Gamma_5 \Rightarrow \Delta_5 \mid \ldots \mid \Gamma_n \Rightarrow \Delta_n$$

Countermodel extraction from saturated hypersequent

- Every component corresponds to a world.
- Formulas in Γ are true, formulas in Δ are false.

$$\Gamma_1 \Rightarrow \Delta_1 \mid \Gamma_2 \Rightarrow \Delta_2 \mid \Gamma_3 \Rightarrow \Delta_3 \mid \Gamma_4 \Rightarrow \Delta_4 \mid \Gamma_5 \Rightarrow \Delta_5 \mid \ldots \mid \Gamma_n \Rightarrow \Delta_n$$

$$A \in \Gamma_i$$

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$A \in \Gamma_i$ $\qquad\qquad\qquad$ $A \in \Delta_k$

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$$A \in \Gamma_i \qquad A \notin \Gamma_j \cup \Delta_j \qquad A \in \Delta_k$$

Countermodel extraction from saturated hypersequent

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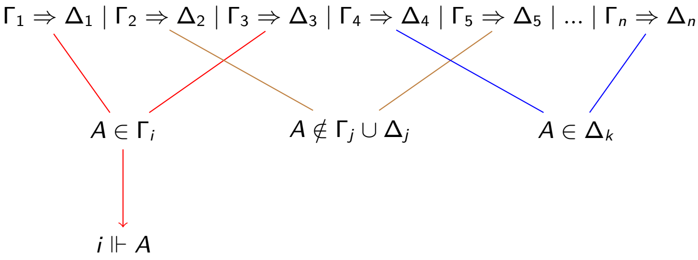

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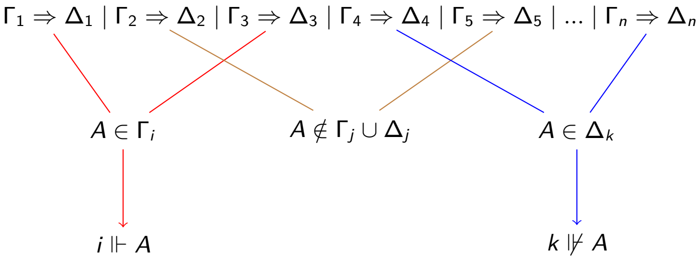

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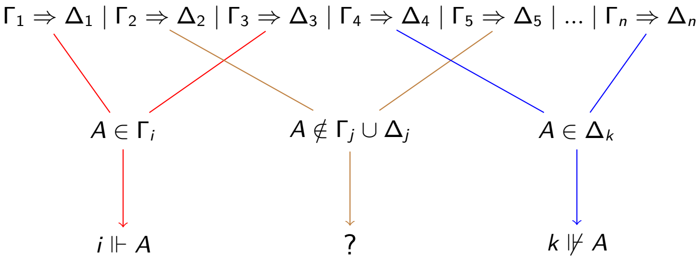

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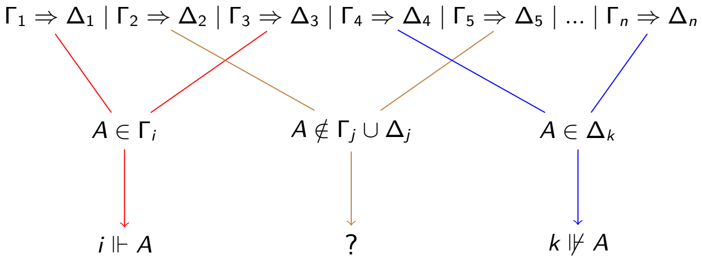

Impossible to determine $[\![A]\!]$.

$\Rightarrow$ Impossible to define directly a neighbourhood model.

Countermodel extraction from saturated hypersequent

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- Formulas in Γ are true, formulas in Δ are false.

1st solution

Saturate with analytic cut:

$$\frac{G \mid \Gamma \Rightarrow \Delta, A \qquad G \mid A, \Gamma \Rightarrow \Delta}{G \mid \Gamma \Rightarrow \Delta} \text{cut}$$

Gain

- Fixes the extension of every subformula
- Constructs a standard neighbourhood model

Loss

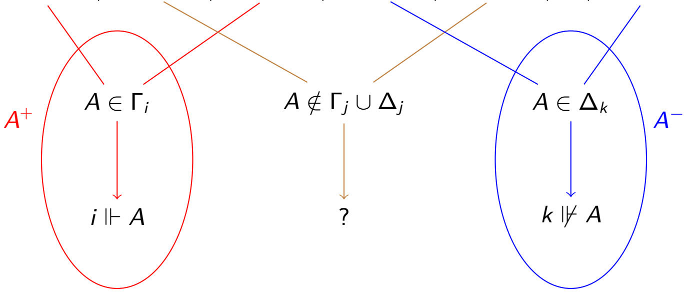
- Strong increase in complexity

Countermodel extraction from saturated hypersequent

- Every component corresponds to a world.
- Formulas in Γ are true, formulas in Δ are false.

2nd solution

$$\Gamma_1 \Rightarrow \Delta_1 \mid \Gamma_2 \Rightarrow \Delta_2 \mid \Gamma_3 \Rightarrow \Delta_3 \mid \Gamma_4 \Rightarrow \Delta_4 \mid \Gamma_5 \Rightarrow \Delta_5 \mid \ldots \mid \Gamma_n \Rightarrow \Delta_n$$



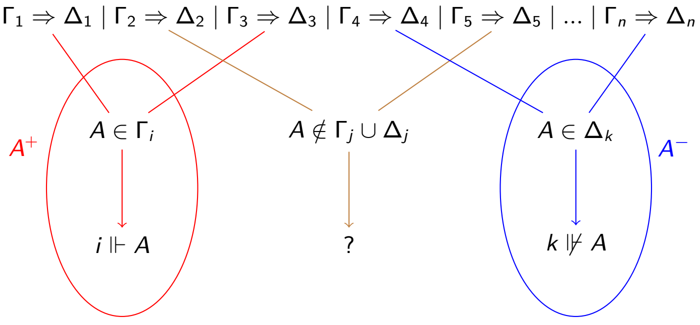
$A^+ \subseteq [\![A]\!]$

$A^- \subseteq [\![\neg A]\!]$

Countermodel extraction from saturated hypersequent

- Every component corresponds to a world.
- Formulas in Γ are true, formulas in Δ are false.

2nd solution



$A^+ \subseteq [\![A]\!]$ $\quad$ $A^- \subseteq [\![\neg A]\!]$

Bi-neighbourhood semantics!

- Every component corresponds to a world.
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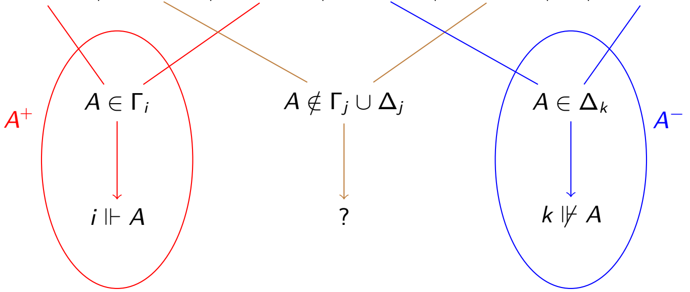
2nd solution

$$\Gamma_1 \Rightarrow \Delta_1 \mid \Gamma_2 \Rightarrow \Delta_2 \mid \Gamma_3 \Rightarrow \Delta_3 \mid \Gamma_4 \Rightarrow \Delta_4 \mid \Gamma_5 \Rightarrow \Delta_5 \mid \dots \mid \Gamma_n \Rightarrow \Delta_n$$



$A^+ \subseteq [\![A]\!]$ $A^- \subseteq [\![\neg A]\!]$

$$\mathbb{E}_i A \in \Gamma_m \quad \rightarrow \quad (A^+, A^-) \in \mathcal{N}_i^{\mathbb{E}}(m)$$

Failure of delegation: Failed proof and bi-neighbourhood countermodel

saturated

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, \langle p\rangle^{\mathbb{C}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p \mid \Rightarrow p$$

$\mathsf{Q}_{\mathbb{C}}$

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, \langle p\rangle^{\mathbb{C}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

$\mathsf{Int}_{\mathbb{E}\mathbb{C}}$

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

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$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

$\mathsf{T}_{\mathbb{E}}$

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

$\mathsf{L}_{\mathbb{E}}$

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

$\mathsf{T}_{\mathbb{E}}$

$$\vdots \qquad \langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p$$

$\mathsf{R}_{\mathbb{E}}$

$$\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p$$

$\mathsf{L}_{\mathbb{E}}$

$$\mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p$$

Failure of delegation: Failed proof and bi-neighbourhood countermodel

$$\mathcal{W} = \qquad\qquad 1 \qquad\qquad 2 \qquad\qquad 3$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, \langle p\rangle^{\mathbb{C}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p \mid \Rightarrow p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, \langle p\rangle^{\mathbb{C}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{Q}_{\mathbb{C}}}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{Int}_{\mathbb{E}\mathbb{C}}}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{Int}_{\mathbb{E}\mathbb{C}}}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{T}_{\mathbb{E}}}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{L}_{\mathbb{E}}}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{T}_{\mathbb{E}}$$

$$\dfrac{\dfrac{\vdots \qquad \langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p}\ \mathsf{R}_{\mathbb{E}}}{\mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p}\ \mathsf{L}_{\mathbb{E}}$$

Failure of delegation: Failed proof and bi-neighbourhood countermodel

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$$\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, \langle p\rangle^{\mathbb{C}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{Int}_{\mathbb{E}\mathbb{C}}$$

$$\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \langle \mathbb{E}_b p\rangle^{\mathbb{C}}_a, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, p, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{Int}_{\mathbb{E}\mathbb{C}}$$

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$$\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \langle p\rangle^{\mathbb{E}}_b, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{L}_{\mathbb{E}}$$

$$\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_b p, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}\ \mathsf{T}_{\mathbb{E}}$$

$$\dfrac{\vdots \qquad \langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p \mid p \Rightarrow \mathbb{E}_b p}{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p}\ \mathsf{R}_{\mathbb{E}}$$

$$\dfrac{\langle \mathbb{E}_b p\rangle^{\mathbb{E}}_a, \mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p}{\mathbb{E}_a\mathbb{E}_b p \Rightarrow \mathbb{E}_a p}\ \mathsf{L}_{\mathbb{E}}$$

- $\mathcal{W} = \{1, 2, 3\} \quad \mathcal{V}(p) = \{1, 2\}$
- $p^+ = \{1, 2\}$; $p^- = \{3\}$; $\mathbb{E}_b p^+ = \{1\}$; and $\mathbb{E}_b p^- = \{2\}$
- $\mathcal{N}^{\mathbb{E}}_b(1) = \mathcal{N}^{\mathbb{C}}_b(1) = \{(p^+, p^-)\} = \{(\{1, 2\}, \{3\})\} \quad \Rightarrow 1 \Vdash \mathbb{E}_b p$
- $\mathcal{N}^{\mathbb{E}}_a(1) = \mathcal{N}^{\mathbb{C}}_a(1) = \{(\mathbb{E}_b p^+, \mathbb{E}_b p^-)\} = \{(\{1\}, \{2\})\} \Rightarrow 1 \Vdash \mathbb{E}_a\mathbb{E}_b p$; $1 \nVdash \mathbb{E}_a p$

Modular extention of **H.ELG**

- Rules of **H.ELG** (formulated with groups)
- Specific rules for groups:

$$\mathsf{F}_{\mathbb{C}}\ \frac{}{G \mid \Gamma, \langle \Sigma \rangle^{\mathbb{C}}_{\emptyset} \Rightarrow \Delta} \qquad \mathsf{Int}^2_{\mathbb{EC}}\ \frac{G \mid \Gamma, \langle \Sigma \rangle^{\mathbb{E}}_{g_1}, \langle \Pi \rangle^{\mathbb{E}}_{g_2}, \langle \Sigma, \Pi \rangle^{\mathbb{C}}_{g_1 \cup g_2} \Rightarrow \Delta}{G \mid \Gamma, \langle \Sigma \rangle^{\mathbb{E}}_{g_1}, \langle \Pi \rangle^{\mathbb{E}}_{g_2} \Rightarrow \Delta}$$

Example: No coalition monotonicity

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textbf{saturated}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \langle p\rangle^{\mathbb{C}}_{\{a\}}, p, \mathbb{E}_{\{a\}}p \Rightarrow \mathbb{E}_{\{a,b\}}p \mid \Rightarrow p}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \langle p\rangle^{\mathbb{C}}_{\{a\}}, p, \mathbb{E}_{\{a\}}p \Rightarrow \mathbb{E}_{\{a,b\}}p}\ \mathsf{Q}_{\mathbb{C}}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, p, \mathbb{E}_{\{a\}}p \Rightarrow \mathbb{E}_{\{a,b\}}p}\ \mathsf{Int}_{\mathbb{E}\mathbb{C}}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \mathbb{E}_{\{a\}}p \Rightarrow \mathbb{E}_{\{a,b\}}p}\ \mathsf{T}_{\mathbb{E}}}{\mathbb{E}_{\{a\}}p \Rightarrow \mathbb{E}_{\{a,b\}}p}\ \mathsf{L}_{\mathbb{E}}$$

$$
\mathcal{W} = \qquad\qquad 1 \qquad\qquad 2
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \langle p\rangle^{\mathbb{C}}_{\{a\}}, p, \mathbb{E}_{\{a\}} p \Rightarrow \mathbb{E}_{\{a,b\}} p \mid \Rightarrow p}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \langle p\rangle^{\mathbb{C}}_{\{a\}}, p, \mathbb{E}_{\{a\}} p \Rightarrow \mathbb{E}_{\{a,b\}} p}\ \mathsf{Q}_{\mathbb{C}}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, p, \mathbb{E}_{\{a\}} p \Rightarrow \mathbb{E}_{\{a,b\}} p}\ \mathsf{Int}_{\mathbb{EC}}}{\langle p\rangle^{\mathbb{E}}_{\{a\}}, \mathbb{E}_{\{a\}} p \Rightarrow \mathbb{E}_{\{a,b\}} p}\ \mathsf{T}_{\mathbb{E}}}{\mathbb{E}_{\{a\}} p \Rightarrow \mathbb{E}_{\{a,b\}} p}\ \mathsf{L}_{\mathbb{E}}
$$

- $\mathcal{W} = \{1, 2\} \quad \mathcal{V}(p) = \{1\}$
- $p^+ = \{1\}$; $p^- = \{2\}$
- $\mathcal{N}^{\mathbb{E}}_{\{a\}}(1) = \{(p^+, p^-)\} = \{(\{1\}, \{2\})\} \qquad \Rightarrow 1 \Vdash \mathbb{E}_{\{a\}} p$
- $\mathcal{N}^{\mathbb{E}}_{\{a,b\}}(1) = \emptyset \qquad \Rightarrow 1 \nVdash \mathbb{E}_{\{a,b\}} p$

Conclusion

Summary

- Hypersequent calculi and bi-neighbourhood semantics for Elgesem's agency logic and Troquard's coalition logic
- Constructive decision procedure: for every formula a proof or a countermodel

To do

- Cover further extensions
- Implementation (HYPNO) style

Open problem

- Proof-theoretic interpolation

T. Dalmonte, C. Grellois, and N. Olivetti. Systèmes de preuve pour les logiques de "Bringing-it-About". JIAF 2021

T. Dalmonte, C. Grellois, and N. Olivetti. Proof systems for the logics of inging-it-about. DEON 2020/21


Thank you!!
Any questions?
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